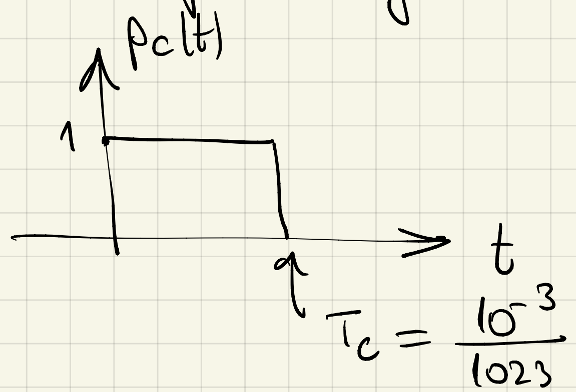
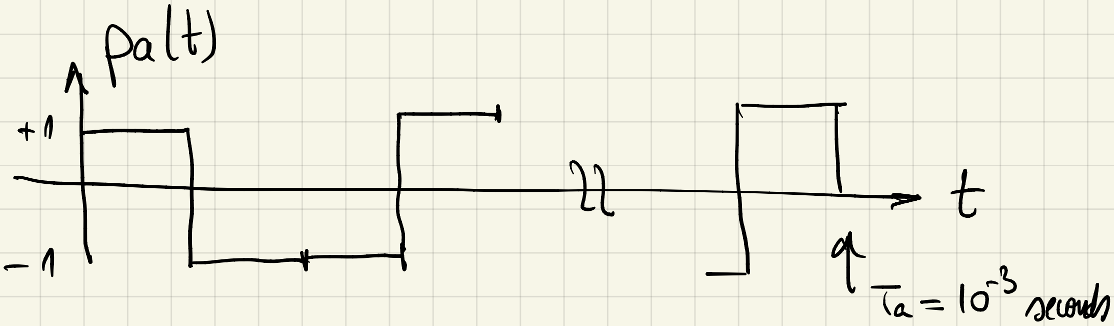
GPS: Decoding the bits

The structure of the signal transmitted by a satellite:

The chip

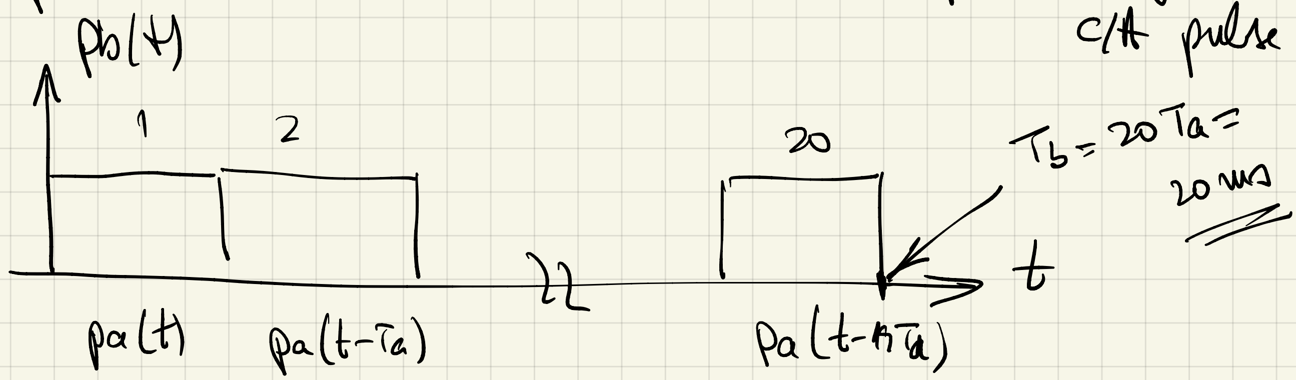


The C/A pulse
(1023 chips)



- unique for each satellite
- C/A code: the corresponding sequence of ±1
  (pseudo-random sequence)

The pulse which carries a bit: 20 repetitions of a C/A pulse



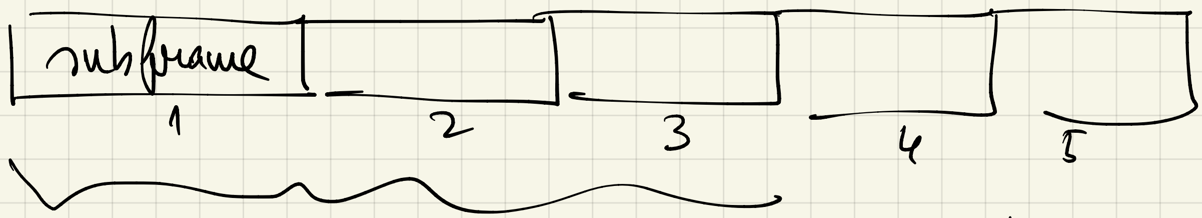
The satellite is sending the i-th bit by sending
$+p_b(t-iT_b)$ or $-p_b(t-iT_b)$

Data structure

Words : consists of 30 bits (600 ms)

Subframe : consist of 10 words (6 s)

Page : consists of 5 subframes (30 s)



↑ Here we find everything we need to determine the position

Clock conventions

t : GPS reference time/clock

t^k : time of satellite k

t_r : time at the receiver of interests

t : is kept by ground stations, and is very precise time (clock)

$t^k = t + \delta t^k$ → the ground stations keep track of it, send it back to the sats, which will then send it down to the receiver

$t_r = t + \delta t_r$

↑ quite "important" variations

From PDC (Principles of Digital Communications) we [know] that a signal can always be written as

$$s(t) = \mathrm{Re}\left\{ e^{j2\pi f_c t} \underbrace{s_E(t)}_{\text{baseband - equivalent}} \right\}$$

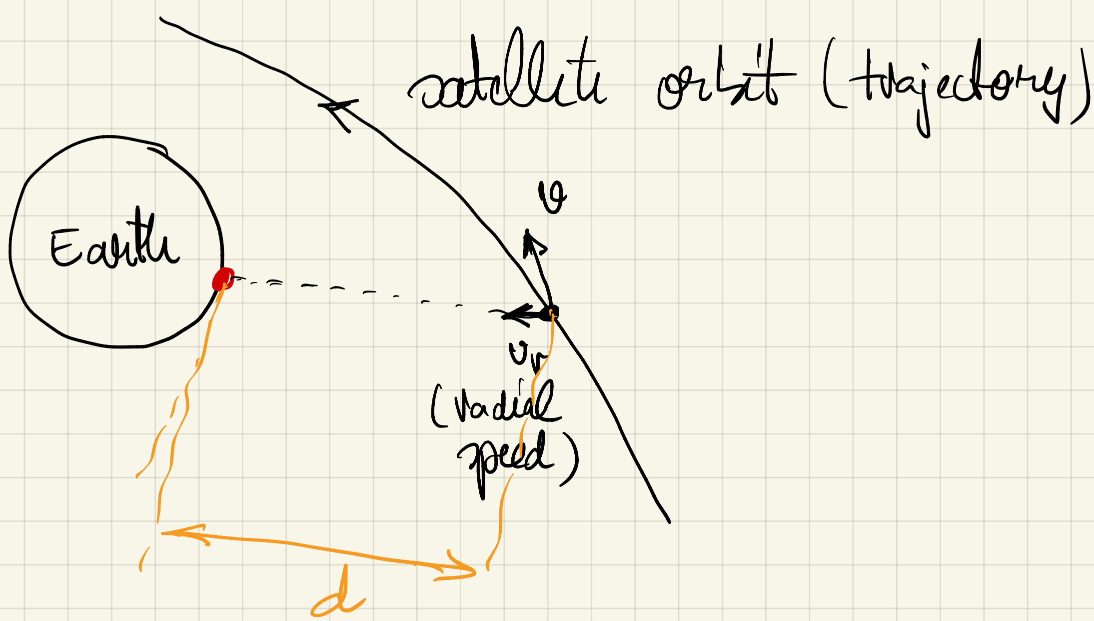
f_c: carrier frequency

The satellite transmits

$$s_E(t) = \sum_i \underbrace{b_i}_{\{\pm 1\},\ i\text{-th bit}} p_b(t - iT_b)$$

$$p_b(\xi) = \sum_{i=0}^{19} p_a(\xi - iT_a)$$

<u>Received signal</u>



Over a short time interval we can approximate that the satellite is moving towards the receiver at a constant speed v_r.

At GPS time t the distance between the receiver and the satellite can be written as

$$d(t) = d_0 - v_r t \quad , \text{ for some } d_0$$

A signal transmitted at GPS time t_1, will reach the receiver at GPS time t_2

$$t_2 = t_1 + \underbrace{\frac{d(t_1)}{c}}_{\text{propagation time}} =$$

$$= t_1 + \frac{d_0 - v_r t_1}{c} = t_1 \left(1 - \underbrace{\frac{v_r}{c}}_{\nu}\right) + \frac{d_0}{c}$$

$$= t_1 (1 - \nu) + \frac{d_0}{c}$$

At GPS time t_1, the satellite clock shows $t^\wedge = t_1 + dt^\wedge$

At GPS time t_2, the receiver clock shows $t_r = t_2 + dt_r$

$$t_r - dt_r = (t^\wedge - dt^\wedge)(1 - \nu) + \frac{d_0}{c}$$

Solve for $t^\wedge$ to find the time showed by the satellite when it transmitted the signal which reached the receiver, at receiver time t_r

$$t^s = \frac{t_r - dt_r - \frac{d_0}{c}}{1-\nu} + dt^s$$

$$= \frac{t_r}{1-\nu} - \underbrace{\left(\frac{dt_r + \frac{d_0}{c}}{1-\nu} - dt^s\right)}_{\tau_0} = \frac{t_r}{1-\nu} - \tau_0$$

Neglecting for now the noise and the signal attenuation, the received signal at receiver time t_r

$$r(t_r) = s(t^s) = s\left(\frac{t_r}{1-\nu} - \tau_0\right)$$

We get a delayed and time-scaled version of the transmitted signal

$$\frac{1}{1-\nu} = \underline{1+\nu} + \nu^2 + \nu^3 + \dots$$

Since $\nu = \frac{v_r}{c} \ll 1$, we can approximate

$$\frac{1}{1-\nu} = 1+\nu$$

$$\Rightarrow \boxed{r(t_r) = s\Big(\underbrace{t_r(1+\nu)}_{\text{time-scaling}} - \underbrace{\tau_0}_{\text{delay}}\Big)}$$

Recall: $s(t) = \mathrm{Re}\left\{ e^{j2\pi f_c t} \underbrace{s_E(t)}_{\text{baseband-equivalent}} \right\}$

$$r(t_r) = \mathrm{Re}\left\{ e^{j2\pi f_c (t_r(1+\nu) - \tau_0)} s_E(t_r(1+\nu) - \tau_0) \right\}$$

$$= \mathrm{Re}\Big\{ \underbrace{e^{j2\pi f_c t_r}\, e^{j2\pi f_c \nu t_r}\, e^{-j2\pi f_c \tau_0}\, s_E\big(t_r(1+\nu)-\tau_0\big)}_{r_E(t_r)} \Big\}$$

$$r_E(t_r) = \underbrace{e^{j2\pi f_c \nu t_r}}_{\substack{\text{Doppler effect} \\ \text{with Doppler frequency} \\ f_d = f_c \nu}}\; \underbrace{e^{-j2\pi f_c \tau_0}}_{\substack{\text{Rotation by a phase} \\ \phi_0 = -j2\pi f_c \tau_0}}\; s_E\big(\underbrace{t_r(1+\nu)}_{\substack{\text{time-scaling} \\ \text{by } 1+\nu}} \underbrace{-\tau_0}_{\text{delay}}\big)$$

- all these above are unknown
- all of them are constant over a short period of time. Need to be reassessed as time goes by

Recall that $\nu = \frac{v_r}{c} \ll 1$

$$t_r(1+\nu) = t_r + \nu t_r \approx t_r$$

Hence $s_E(t_r(1+\nu) - \tau_0) \simeq s_E(t_r - \tau_0)$

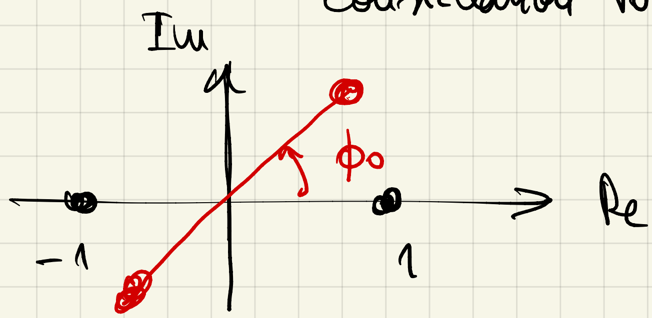
So we obtain:

$$\boxed{r_E(t_r) = e^{j2\pi f_d t_r}\, e^{j\phi_0}\, s_E(t_r - \tau_0)}$$

- unknown
- they are independent

$$s_E(t) = \sum_i b_i \, p_b(t - iT_b)$$

$$e^{j\phi_0} s_E(t) = \sum_i \underbrace{e^{j\phi_0} b_i}_{\text{Constellation rotation}} p_b(t - iT_b)$$



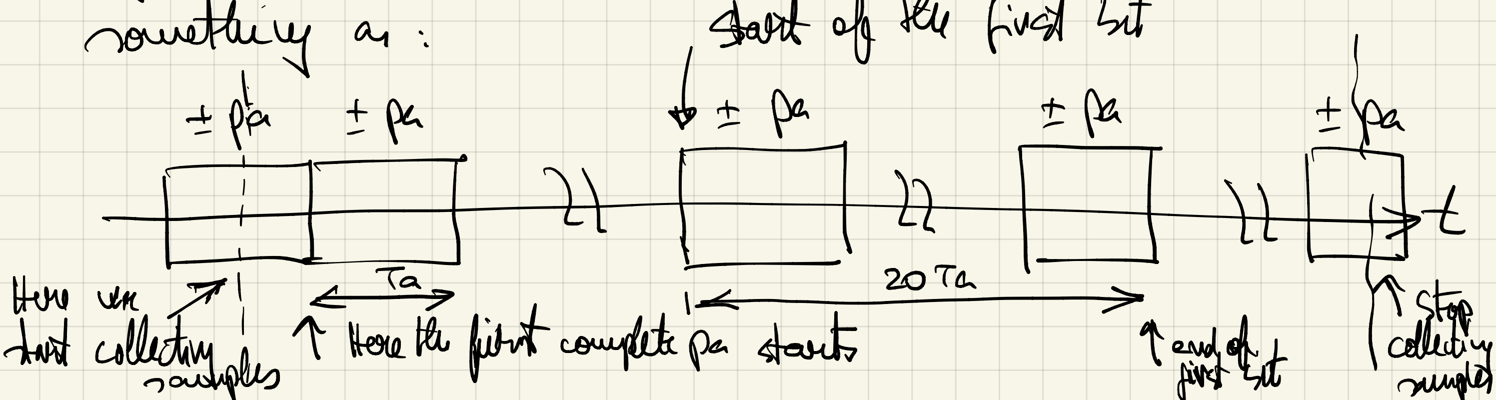
We do not estimate ϕ_0. Instead, we do differential decoding.

The idea: we assume that the first received bit is $+1$. Then, for the next bit, if the phase difference is $\simeq 0$ $\Rightarrow$ the bit is the same as the previous. If the phase difference $\simeq \pi$ $\Rightarrow$ we have $-$previous bit.

At the end, if the first bit was indeed -1, we get all the bits fliped (we can detect this using preambles inverted in the subframes).

This week: we estimate f_d and τ_0

If we neglect the Doppler and the noise, we receive something as:

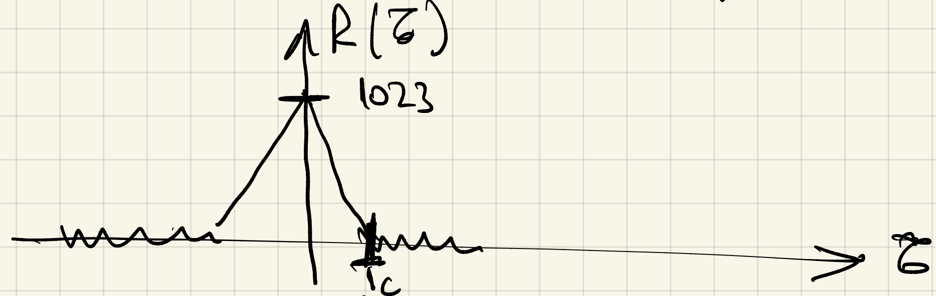


Goal of this week:

- find the beginning of the first C/A pulse into the received samples
- estimate f_d

Key property which helps finding the parameters above:

$p_a(\xi)$ has a sharp self-similarity function



$$\boxed{R(\tau) = \int p_a(\xi+\tau)\, p_a^*(\xi)\, d\xi}$$

How to:

- find a coarse estimate of the Doppler shift
- find the start of a (the first) C/A pulse in the received samples

We loop over the satellites (1... 39)

- loop over a coarse grid of f_d values in the range $[-5, 5]$ kHz
- correlate the sequence

$$r(nT_s)\, e^{-j2\pi f_d n T_s} \qquad n = 1, 2 \ldots$$

received sequence of samples

n-th sample

tentative correction of f_d

with the C/A code of that satellite

- for each of them store the maximum value of the correlation (absolute value), and it occurs (this will give us τ_0)

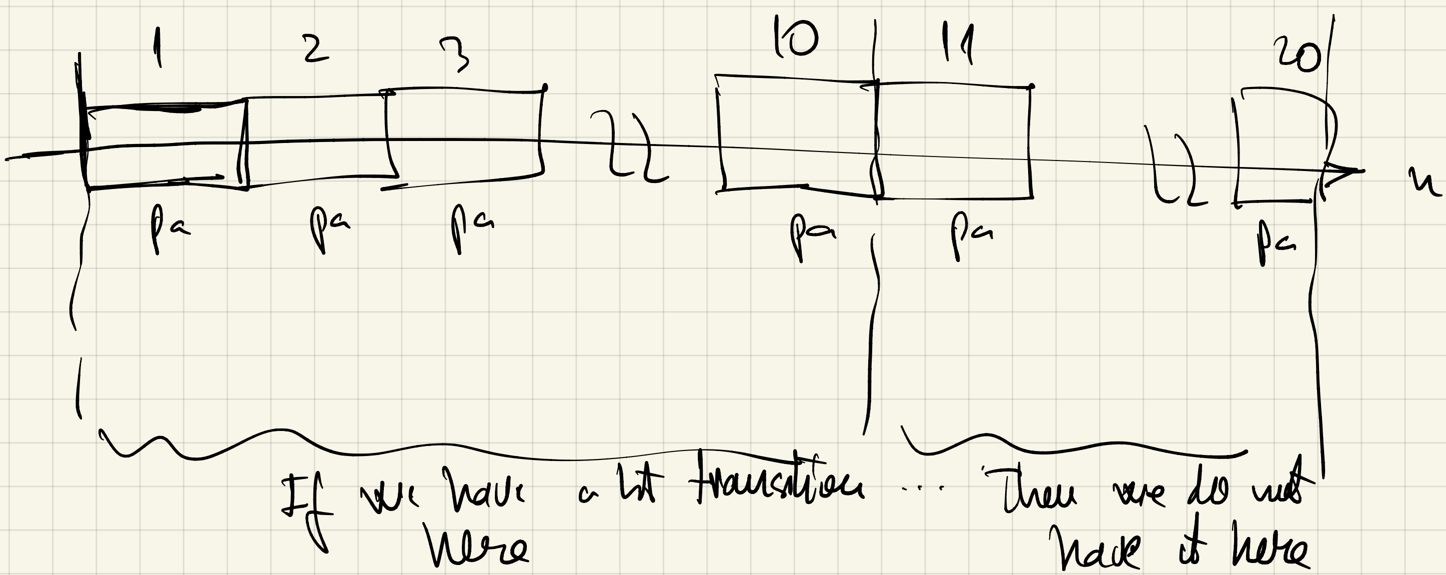
  candidate fd

Outside the loop over the satellites, we order them (the sats) in decreasing order of the correlations values, and we select the first 6 ones.

At the end of the loop over the doppler (for each of these selected sats) we have:

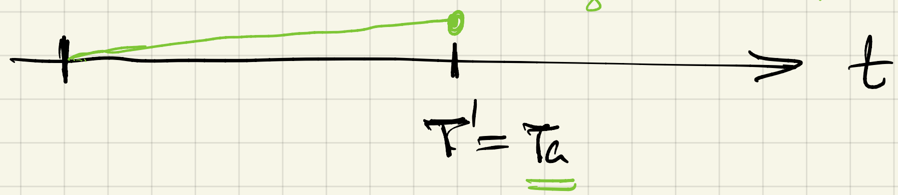
- a coarse estimate of fd
- an estimate of the position where the first C/A code starts.

We need to refine the estimation of the fd.



How to choose the step of the fd :

angle due to Doppler. Should not exceed $\frac{2\pi}{10}$ at T'



Over this time interval, we compare

$$a(t)\, e^{j2\pi f_d t} \quad \text{with} \quad a(t)$$

We will not find a match unless

$$2\pi f_d T' < \text{some small angle (let us say } \tfrac{2\pi}{10}\text{)}$$

$$T' = 10^{-3} \qquad 2\pi f_d T' < \frac{2\pi}{10}$$

$$f_d < \frac{1}{10 \cdot T'} = \frac{1}{10^{-2}} = 100 \text{ Hz.}$$

coarse step for f_d

Fine estimate: we choose $\frac{\text{coarse step}}{10} = 10$ Hz